Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Coefficient of $x^4 = 15$	**B1**	Condone inclusion of x^4. Can be seen as part of an expansion.
	Coefficient of $x^2 = 240a^2$	**B1**	Condone inclusion of x^2. Can be seen as part of an expansion.
	'*Their* 240'a^2 – '*their* 15'	**M1**	Forming an equation of the form $pa^2 = q$, where p and q are constants. Condone inclusion of powers of x as long as they then disappear.
	$a = \frac{1}{4}$ or 0.25	**A1**	OE Do not condone extra 'answer' of $-\frac{1}{4}$,or allow $\sqrt{\frac{1}{16}}$ or similar.
		4	

Question	Answer	Marks	Guidance
2	$r = 0.8$	**B1**	OE
	$a = 12.5$	**B1**	OE
	$S_\infty = 12.5 \div (1 - 0.8)$	**M1**	Using $\frac{a}{1-r}$ with '*their a*' and '*their r*' but $\lvert r \rvert$ must be < 1.
	$S_\infty = \frac{125}{2}, 62\frac{1}{2}$ or 62.5	**A1**	$\dfrac{12\frac{1}{2}}{\frac{1}{5}}$ or similar **does not** get A1.
		4	

Question	Answer	Marks	Guidance
3	$[y=]\left\{\dfrac{3(4x-7)^{\frac{3}{2}}}{\frac{3}{2}\times 4}\right\}+\left\{-\dfrac{4}{\frac{1}{2}}x^{\frac{1}{2}}\right\}\left[\Rightarrow \dfrac{1}{2}(4x-7)^{\frac{3}{2}}-8x^{\frac{1}{2}}\right][+c]$	**B1 B1**	Marks can be awarded for correct unsimplified expressions ISW.
	$\dfrac{5}{2}=\dfrac{1}{2}(9)^{\frac{3}{2}}-8\times 4^{\frac{1}{2}}+c \quad [\Rightarrow c=5]$	**M1**	Using $(4, \frac{5}{2})$ in an integrated expression (defined by at least one correct power) including + *c*.
	$y=\dfrac{3}{6}(4x-7)^{\frac{3}{2}}-8x^{\frac{1}{2}}+5$.	**A1**	Condone *c* = 5 as their final line if either *y* = or f(*x*) = seen elsewhere in the solution. Coefficients must not contain unresolved double fractions.
		4	

Question	Answer	Marks	Guidance
4(a)	$2\times 6k=k+k+6$ or $6k-k=k+6-6k$ or $2d = 6$ leading to $d = 3$, $\therefore 6k-3=k$	**B1**	OE A correct equation in *k* only. Can be implied by correct final answer.
	$k=\dfrac{6}{10}$ or 0.6	**B1**	OE
		2	

Question	Answer	Marks	Guidance
4(b)	$d = 3$	**B1**	Correct value of *d* can be implied by a correct final answer. Working may be seen in part **(a)** but must be used in **(b)**.
	$S_{30} = \frac{30}{2}\left(2 \times \text{'their } k\text{'} + 29 \times \text{'their } d\text{'}\right)$	**M1**	It needs to be clear that the candidate is using a correct sum formula. There is no requirement to check the candidates working for *d* but it must be clearly identified.
	$S_{30} = 1323$	**A1**	ISW if corrected to 1320.
		3	

Question	Answer	Marks	Guidance
5(a)	$4\times 0^2 - 0 + \frac{1}{2}k^2 = 0 - a$	M1	Equating the equations of curve and line and substituting $x = 0$. Condone slight errors e.g. ± sign errors.
	$4\times\left(\frac{3}{4}\right)^2 - \frac{3}{4}k + \frac{1}{2}k^2 = \frac{3}{4} - a$	M1	Equating the equations of curve and line and substituting $x = \frac{3}{4}$. Condone slight errors e.g. ± sign errors.
	$k = 2,\ \ a = -2$	A1 A1	WWW
	Alternative method for question 5(a)		
	$(x-0)\left(x-\frac{3}{4}\right) = 0$ or $x(4x-3) = 0$ [$\Rightarrow 4x^2 - 3x = 0$]	*M1	Use 0, $\frac{3}{4}$ to form a quadratic equation. **Do not allow** $(x+0)\left(x+\frac{3}{4}\right) = 0$.
	$4x^2 - kx + \frac{1}{2}k^2 = x - a$ leading to $4x^2 - (k+1)x + \frac{1}{2}k^2 + a[=0]$	DM1	Equating the equations of curve and line and rearranging so that terms are all on same side. Condone slight errors e.g. ± sign errors.
	$k = 2,\ \ a = -2$	A1 A1	WWW
	Alternative method for question 5(a)		
	$-\frac{b}{a} = \frac{3}{4} + 0$ and $\frac{c}{a} = 0 \times \frac{3}{4}$	*M1	Using sum and product of roots. Condone ± sign errors.
	$\frac{k+1}{4} = \frac{3}{4}$ and $\frac{\frac{1}{2}k^2 + a}{4} = 0$	DM1	Equating the equations of curve and line and equating to $\frac{3}{4}$ and 0.
	$k = 2,\ \ a = -2$	A1 A1	WWW
		4	

Question	Answer	Marks	Guidance
5(b)	$4x^2 - kx + \frac{1}{2}k^2 = x + \frac{7}{2} \Rightarrow 4x^2 - kx - x + \frac{1}{2}k^2 - \frac{7}{2}[=0]$	*M1	OE Substitute $a = -\frac{7}{2}$ and rearrange so that terms are all on same side, condone ± sign errors. Watch for multiples.
	$(k+1)^2 - 4 \times 4\left(\frac{1}{2}k^2 - \frac{7}{2}\right)$	*DM1	Use of $b^2 - 4ac$ with the coefficients from *their* 3-term quadratic. Both coefficients 'b' and 'c' must consist of two components.
	$\Rightarrow 7k^2 - 2k - 57$	A1	OE
	$(k-3)(7k+19)$ or other valid method	DM1	Factorising or use of the formula or completing the square. Must be evidence of an attempt to solve for this mark. Dependent upon both previous method marks.
	$k = 3,\ k = -\frac{19}{7}$	A1	OE e.g. AWRT -2.71. No ISW if inequalities used. **SC:** If second DM1 not scored, **SC B1** available for correct final answers.
	Alternative method for question 5(b)		
	$8x - k = 1$ **and** $4x^2 - kx + \frac{1}{2}k^2 = x + \frac{7}{2}$	*M1	Equating gradients and equating line and curve.
	$4x^2 - (8x-1)x + \frac{1}{2}(8x-1)^2 = x + \frac{7}{2}$ **or** $4\left(\frac{k+1}{8}\right)^2 - k\left(\frac{k+1}{8}\right) + \frac{1}{2}k^2 = \frac{k+1}{8} + \frac{7}{2}$	*DM1	Forming an equation in x or k only.
	$28x^2 - 8x - 3$ or $7k^2 - 2k - 57$	A1	OE A correct 3 term quadratic in x or k only.
	$(14x+3)(2x-1)$ or $(k-3)(7k+19)$ or other valid method	DM1	OE Factorising or use of the formula or completing the square. Must be evidence of an attempt to solve for this mark. Dependent upon both previous method marks.

Question	Answer	Marks	Guidance
5(b)	$k = 3,\ k = -\dfrac{19}{7}$	**A1**	OE e.g. AWRT -2.71. No ISW if inequalities used. **SC:** If second DM1 not scored, **SC B1** available for correct final answers.
		5	

Question	Answer	Marks	Guidance
6	Line meets curve when: $2x+2=5x^{\frac{1}{2}}$ leading to $2x - 5x^{\frac{1}{2}}+2[=0]$ **or** $4x^2+8x+4=25x$ leading to $4x^2-17x+4[=0]$ **or** $x=\dfrac{y^2}{25}$ leading to $2y^2-25y+50[=0]$	**M1**	Equating line and curve and rearranging so that terms are all on same side, condone sign errors, and making a valid attempt to solve by factorising, using the formula or completing the square. Factors are:$(2x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-2)$, $(4x-1)(x-4)$ and $(2y-5)(y-10)$.
	$x=\dfrac{1}{4},\ x=4$	**A1**	**SC:** If M1 not scored, **SC B1** available for correct answers, could just be seen as limits.
	Area = $\int 5x^{\frac{1}{2}}-(2x+2)\mathrm{d}x = \int 5x^{\frac{1}{2}}-2x-2\ \mathrm{d}x$	***M1**	Intention to integrate and subtract areas. Condone missing brackets and/or subtraction wrong way around.
	$=\left[\dfrac{10}{3}x^{\frac{3}{2}}-x^2-2x\right]_{\frac{1}{4}}^{4} = \left(\left(\dfrac{10}{3}\times 8-16-8\right)-\left(\dfrac{10}{3}\times\dfrac{1}{8}-\dfrac{1}{16}-\dfrac{1}{2}\right)\right)$	**DM1**	Integrating($kx^{\frac{3}{2}}$ seen) and substituting '*their* points of intersection' (but limits need to be found, not assumed to be 0 and something else).
	$\dfrac{45}{16}$ or $2\dfrac{13}{16}$ or 2.8125	**A1**	OE exact answer. Condone $-\dfrac{45}{16}$ if corrected to $\dfrac{45}{16}$. A0 for inclusion of π. **SC:** If *M1 DM0 scored, **SC B1** available for correct answer.

Question	Answer	Marks	Guidance
6	**Alternative method for question 6**		
	Line meets curve when: $2x+2=5x^{\frac{1}{2}} \Rightarrow 2x - 5x^{\frac{1}{2}}+2[=0]$ **or** $4x^2+8x+4=25x \Rightarrow 4x^2-17x+4[=0]$ **or** $x=\frac{y^2}{25} \Rightarrow 2y^2-25y+50[=0]$	**M1**	Equating line and curve and rearranging so that terms are all on same side, condone sign errors, and making a valid attempt to solve by factorising, using the formula or completing the square. Factors are:$(2x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-2)$, $(4x-1)(x-4)$ and $(2y-5)(y-10)$.
	$x=\frac{1}{4}, x=4$	**A1**	**SC:** If M1 not scored, **SC B1** available for correct answers, could just be seen as limits.
	Area = $\int 5x^{\frac{1}{2}}\mathrm{d}x$ – $\{\int(2x+2)dx$ or area of trapezium$\}$	***M1**	Intention to integrate and subtract areas. Or integrate curve and subtract area of trapezium.
	$\left[\frac{10}{3}x^{\frac{3}{2}}\right]_{\frac{1}{4}}^{4} - \left\{\left[x^2+2x\right]_{\frac{1}{4}}^{4} \textit{ or } \frac{1}{2}(\text{sum of ‘}\textit{their } y \text{ values’})\text{‘}\textit{their}\frac{15}{4}\text{’}\right\}$ $=\left(\left(\frac{10}{3}\times 8\right)-\left(\frac{10}{3}\times\frac{1}{8}\right)\right)-\left\{\left((16+8)-\left(\frac{1}{16}+\frac{1}{2}\right)\right)\textit{or } \frac{1}{2}\left(\frac{5}{2}+10\right)\frac{15}{4}\right\}$	**DM1**	Integrating ($kx^{\frac{3}{2}}$ seen) and substituting ‘*their* points of intersection’ (but limits need to be found, not assumed to be 0 and something) or a trapezium using the correct formula (‘*their* $\frac{15}{4}$’ must be ‘*their* 4’ – ‘*their* $\frac{1}{4}$’ **but not 0**).
	$\frac{45}{16}$ or $2\frac{13}{16}$ or 2.8125	**A1**	OE exact answer. Condone $-\frac{45}{16}$ if corrected to $\frac{45}{16}$. A0 for inclusion of π. **SC:** If *M1 DM0 scored, **SC B1** available for correct answer.
		5	

Question	Answer	Marks	Guidance
7(a)	$\left[A\hat{O}B=\right]\ \frac{2}{10}$	**B1**	OE Sight of 0.2 from $s = r\theta$ but $10\theta = 2$ is not enough. ISW if $\frac{2}{10} = \frac{\pi}{5}$.
	$[B\hat{O}C=]\frac{5\pi+6}{30}$ or $\frac{1}{6}\pi + 0.2$	**B1**	OE e.g. 0.724^c AWRT or 41.5 degrees AWRT. But not $\frac{2+\frac{5\pi}{3}}{10}$ – fraction within a fraction. ISW incorrect simplifications.
	Alternative method for question 7(a)		
	OR [Arc AC =] $\frac{10\pi}{6}$ or $[\text{Arc } BC=]\frac{10\pi}{6} + 2$ or 7.2	**B1**	AWRT. Sight of $\frac{10\pi}{6}$ or 5.2 or 7.2.
	$[B\hat{O}C=]\frac{5\pi+6}{30}$ or $\frac{1}{6}\pi + 0.2$	**B1**	OE e.g. 0.724^c AWRT or 41.5 degrees AWRT. But not $\frac{2+\frac{5\pi}{3}}{10}$ – fraction within a fraction. ISW incorrect simplifications.
		2	

Question	Answer	Marks	Guidance
7(b)	$[BP] = 10\sin\left(\frac{5\pi+6}{30}\right)$ and $[OP] = 10\cos\left(\frac{5\pi+6}{30}\right)$ [= 6.6208…] and [= 7.494…] **OR** $[BP] = 10\sin\left(\frac{5\pi+6}{30}\right)$ and $[O\hat{B}P] = \left(\frac{5\pi-3}{15}\right)$ [= 6.6208…] and [= 0.84719…]	**M1**	OE Any correct method for **both** lengths, for *their* angle BOC (which may have been incorrectly 'simplified' but not 0.2) or length BP and $O\hat{B}P$. May be seen as part of $\frac{1}{2}ab\sin C$. Sight of correct method enough. Can be implied by the next A1.
	Area of $\Delta OBP = \frac{1}{2}\times 10\sin\left(\frac{5\pi+6}{30}\right)\times 10\cos\left(\frac{5\pi+6}{30}\right)$ or $\frac{1}{2}\times 10\times 10\sin\left(\frac{5\pi+6}{30}\right)\times\sin\left(\left(\frac{5\pi-3}{15}\right)\right)$ [=24.809]	**A1**	OE Can be implied by any answer in range (24.7, 24.9) or a final answer in the range (11.3, 11.5) WWW.
	$[\text{Sector } BOC] = \frac{1}{2}\times 10^2\times their\ \left(\frac{5\pi+6}{30}\right)$ $\left[= 50\left(\frac{5\pi+6}{30}\right) = 36.1799...\right]$	**M1**	Use of $\frac{1}{2}r^2\theta$ with *their* angle BOC (may have been incorrectly 'simplified' but not 0.2).
	Area of region *BPC* = 11.4	**A1**	CAO
		4	

Question	Answer	Marks	Guidance
8(a)	$1+1+a+b-12=0[\Rightarrow a+b=10]$ $4+36+2a-6b-12=0[\Rightarrow 2a-6b=-28]$	**B1 B1**	B1 for each equation. Allow unsimplified. Can be implied by correct values for *a* and *b*.
	$a=4, b=6$	**B1**	
	Centre is $\left(-\frac{their\ a}{2}, -\frac{their\ b}{2}\right)$ $[-2,-3]$	**B1 FT**	Or $x=-2, y=-3$
		4	

Question	Answer	Marks	Guidance
8(b)	Gradient of AC is $\frac{1-their\ y}{1-their\ x}$ [= $\frac{1--3}{1--2}=\frac{1+3}{1+2}=\frac{4}{3}$]	*M1	Using *their* centre correctly.
	Gradient of tangent is $=\frac{-1}{their\ \frac{4}{3}}\left[=-\frac{3}{4}\right]$	A1 FT	Use of $m_1m_2=-1$ to obtain the gradient of the tangent.
	Equation: $y-1=\text{'}their -\frac{3}{4}\text{'}(x-1)$ or $y=-\frac{3}{4}x+\frac{7}{4}$	DM1	Using $(1,1)$ with *their* gradient of the tangent at *A*.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	A1	
	Alternative method for question 8(b)		
	$2x+2y\frac{dy}{dx}+4+6\frac{dy}{dx}=0$	*M1	Implicit differentiation with at least one *y* term differentiated correctly.
	$8\frac{dy}{dx}=-6\Rightarrow\frac{dy}{dx}=-\frac{6}{8}$	A1	
	Equation: $y-1=\text{'}their -\frac{3}{4}\text{'}(x-1)$ or $y=-\frac{3}{4}x+\frac{7}{4}$	DM1	Using $(1,1)$ with *their* gradient of the tangent at *A*.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	A1	
	Alternative method for question 8(b)		
	$\frac{dy}{dx}=\frac{1}{2}\{25-(x+2)^2\}^{-\frac{1}{2}}(-2x-4)$	*M1	Rearranging to form $y=$ and differentiating using the chain rule.
	$\frac{dy}{dx}=\frac{1}{2}(25-9)^{-\frac{1}{2}}(-6)=-\frac{6}{8}$	A1	

Question	Answer	Marks	Guidance
8(b)	Equation: $y-1=$ '*their* $-\frac{3}{4}$'$(x-1)$ or $y=-\frac{3}{4}x+\frac{7}{4}$	**DM1**	Using $(1,1)$ with *their* gradient of the tangent at *A*.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	**A1**	
		4	

Question	Answer	Marks	Guidance
9(a)	$\frac{dy}{dx}=\{3\}+\left\{-4\times\frac{1}{2}(3x+1)^{-\frac{1}{2}}\times3\right\}\left[=3-6(3x+1)^{-\frac{1}{2}}\right]$	**B1 B1**	Correct differentiation of $3x+1$ and no other terms and correct differentiation of $-4(3x+1)^{\frac{1}{2}}$. Accept unsimplified.
	$\left[\frac{d^2y}{dx^2}=\right]-\frac{1}{2}\times-6(3x+1)^{-\frac{3}{2}}\times3$ $[=9(3x+1)^{-\frac{3}{2}}]$	**B1**	WWW. Accept unsimplified. Do not award if $\frac{dy}{dx}$ is incorrect.
		3	
9(b)	$\frac{dy}{dx}=0$ leading to $3-6(3x+1)^{-\frac{1}{2}}=0$	**M1**	Setting *their* $\frac{dy}{dx}$ = 0.
	$(3x+1)^{\frac{1}{2}}=2 \Rightarrow 3x+1=4$ leading to $x=1$	**A1**	CAO – do not ISW for a second answer.
	$y=-4$ [coordinates (1, −4)]	**A1**	Condone inclusion of second value from a second answer.
	$\frac{d^2y}{dx^2}=9(3\times1+1)^{-\frac{3}{2}}=\frac{9}{8}$ or > 0 so minimum	**A1**	Some evidence of substitution needed but $\frac{d^2y}{dx^2}$. Do not award if $\frac{d^2y}{dx^2}$ is incorrect or wrongly evaluated. Accept correct consideration of gradients either side of $x=1$.
		4	

Question	Answer	Marks	Guidance
10(a)	$x \neq 1$ or $x < 1,\ x > 1$ or $(-\infty, 1), (1, \infty)$ $[x \in \mathbb{R}]$	**B1**	Must be x not $f^{-1}(x)$ or y. Do not accept $1 < x < 1$.
		1	
10(b)	$y = \frac{2x+1}{2x-1}$ leading to $(2x-1)y = 2x+1$ leading to $2xy - y = 2x+1$	***M1**	Setting *y* = , removing fraction and expanding brackets.
	$2xy - 2x = y + 1$ leading to $2x(y-1) = y+1$ leading to $x = \frac{y+1}{2(y-1)}$	**DM1**	Reorganising to get *x* =. Condone ± sign errors only.
	$[f^{-1}(x)] = \frac{x+1}{2(x-1)}, \frac{x+1}{x-1} \times \frac{1}{2}$ or $\frac{1}{x-1} + \frac{1}{2}$	**A1**	OE. Must be in terms of *x*. Do not allow $\frac{x+1}{x-1} \div 2$.
		3	
10(c)	$(their\ f^{-1}(3))$ leading to $(their\ f^{-1}(3))^2 + 4$ $[f^{-1}(3) = 1, 1 + 4 =]$	**M1**	Correct order of operations and substitution of *x* = 3 needed.
	5	**A1**	
		2	
10(d)	Sight of 'not one to one' or 'many to one' or 'one to many'	**B1**	Any reason mentioning 2 values, or + and — , such as: square root gives 2 values or horizontal line test crosses curve twice or 2 values because of turning point or 2 values because it is a quadratic.
		1	

Question	Answer	Marks	Guidance
10(e)	$f(x) = 1 + \frac{2}{2x-1} = \frac{2x-1}{2x-1} + \frac{2}{2x-1} = \frac{2x+1}{2x-1}$	**B1**	AG Do not condone equating expressions and verification.
	$f'(x) = -4(2x-1)^{-2}$ or $2(2x-1)^{-1} + \left\{-(2x+1)2(2x-1)^{-2}\right\}$ or $\frac{(2\boldsymbol{x}-1)2 - 2(2\boldsymbol{x}+1)}{(2\boldsymbol{x}-1)^2}$	***M1**	For $k(2x-1)^{-2}$ and no other terms or correct use of the product or quotient rule then ISW.
	Gradient $m = -4$	**A1**	Differentiation must have clearly taken place.
	Equation of tangent is $y - 3 = -4(x-1)$ $[\Rightarrow y = -4x + 7]$	**DM1**	Using (1, 3) in the equation of a line with *their* gradient.
	Crosses axes at $\left(\frac{7}{4}, 0\right)$ and $(0, 7)$	**A1 FT**	SOI from *their* straight line or by integration from 0 to '*their* 7/4'.
	[Area =] $\frac{49}{8}$	**A1**	OE e.g. 6.13 AWRT. If M0 A0 DM0, **SC B2** available for correct answer.
		6	

Question	Answer	Marks	Guidance
11(a)	$4\cos^4 x + \cos^2 x - 3 = 0 \Rightarrow (4\cos^2 x - 3)(\cos^2 x + 1) = 0$	**M1**	Attempt to solve 3 term quartic (or quadratic in another variable).
	$\Rightarrow [\cos^2 x =]\frac{3}{4}$ $[\cos^2 x = -1]$	**A1**	If M0 scored then **SC B1** is available for sight of $\frac{3}{4}$ [and -1].
	$\Rightarrow \cos x = [\pm]\sqrt{their\,\frac{3}{4}}$ **OE** $\left[= \pm\frac{\sqrt{3}}{2}\right]$	**M1**	Square rooting '*their* $\cos^2 x$'. Allow without ±. May be implied by correct final answer(s). Ignore $\sqrt{-1}$.
	$[x =]\ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	**A1** **A1 FT**	Dependent on preceding M1 only. Exact answers needed. A1 for any 2 correct answers A1 A1 for 4 correct answers and no others inside the range $0 \leqslant x \leqslant 2\pi$ A0 A1 FT can be awarded for two exact answers that are $2\pi -$ '*their* $\frac{\pi}{6}$ and $\frac{5\pi}{6}$' within the range $0 \leqslant x \leqslant 2\pi$.
			SC: If all 4 answers given in degrees (30, 150, 210, 330) or non-exact (AWRT 0.524, 2.62, 3.67, 5.76 or 0.167π, 0.833π, 1.17π, 1.83) and no others then **SC B1**.
		5	

Question	Answer	Marks	Guidance
11(b)	$\cos^2 x = \dfrac{-1-\sqrt{1+16k}}{8} < 0$ [∴ no solutions].	**B1**	State that this root is less than 0, needs to be linked to $\cos^2 x$. Can be achieved by substituting a value for $k \geqslant 0$.
	$[\cos^2 x] = \dfrac{-1\pm\sqrt{1+16k}}{8}$	***M1**	Must use quadratic formula. Allow any value of k **but not** ±3. Condone + rather than ±.
	Substituting $k = 5$ and obtain 1 from the formula	**DM1**	Or argue logically if $k > 5 \Rightarrow 1+16k > 81 \Rightarrow >1$.
	$\cos^2 x = 1$ or $\cos^2 x >$ or $\geqslant 1$	**A1**	Needs to be linked to $\cos^2 x$.
	Concluding statement having considered both ± cases. ∴ no solutions	**A1**	Dependent upon all previous marks having been scored.
	Alternative method for question 11(b)		
	$\cos^2 x = \dfrac{-1-\sqrt{1+16k}}{8} < 0$ [∴ no solutions].	**B1**	State that this root is less than 0, needs to be linked to $\cos^2 x$. Can be achieved by substituting a value for $k \geqslant 0$.
	$[\cos^2 x] = \dfrac{-1\pm\sqrt{1+16k}}{8}$	***M1**	Must use quadratic formula. Allow any value of k **but not** ±3. Condone + rather than ±.
	$\dfrac{-1+\sqrt{1+16k}}{8} * 1 \Rightarrow -1+\sqrt{1+16k} * 8 \Rightarrow 1+16k * 81$	**DM1**	$*$ represents any inequality or =.
	$k * 5$	**A1**	$*$ represents any inequality or =.
	Concluding statement having considered both ± cases. ∴ no solutions	**A1**	Dependent upon all previous marks having been scored.
		5	